Mathematics Assessment Exam

Algebra

1. Compute this complex number operation $(3 + 4i)(5 + 3i)$. Which of the following is the correct result expressed in the form $a + bi$?

   A. $15 + 29i$

   B. $15 + 12i$

   C. $3 + 29i$

   D. $8 + 16i$

2. Which of the following properly uses the hierarchy of operations?

   A. $12 \div 5 - 2 + 3 = 12 \div (5 - 2) + 3 = 12 \div 3 + 3 = 7$

   B. $20 - 5 \cdot 2 + 4 = 15 \cdot 2 + 4 = 34$

   C. $20 \div 5 - 2 \cdot 4 = 4 - 2 \cdot 4 = 8$

   D. $20 \div 5 - 2 \cdot 3 = 4 - 6 = -2$

3. Two buses leave a station at intervals of 6 minutes and 10 minutes, respectively. If two buses leave together at 9:00 AM, when will two buses leave the station together again?

   A. 9:16 AM

   B. 9:30 AM

   C. 10:20 AM

   D. 9:10 AM

4. What is the value of k in the following expression:

$$\frac{32^2 \cdot 64 \cdot 4^4}{8^2 \cdot \sqrt{256} \cdot 16^2} = 2^k$$

   A. $k = 2$

   B. $k = 4$

   C. $k = 6$

   D. None of the other solutions are correct

5. Given the following intervals $A = \{x \in \mathbb{R} \mid x > 3\}$, $B = [1, 6]$, what is the value of the intersection of A and B, $A \cap B$? *Hint: Intersection means all the points both in A and B.*

   A. $(3, 6)$

   B. $(3, 6]$

   C. $[1, 3)$

   D. $[1, \infty)$

6. A polynomial $P(x)$ has degree 3, leading coefficient 1 and its roots are $\{1, 2, 5\}$. What is the sum of all its coefficients?

   A. 8

   B. 1

   C. -8

   D. 0

Linear Algebra

7. Consider the vectors $\vec{u} = (1, 2, 3)$ and $\vec{v} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. What statement is true about the vector $\vec{w} = 2\vec{u} - \vec{v}$?

    A. The length of the vector is $\sqrt{12}$.

    B. The length of the vector $\sqrt{160}$.

    C. Vector $(0, 12, 20)$ is perpendicular to $\vec{w}$.

    D. Vector $(5, 2, -6)$ is perpendicular to $\vec{w}$.

8. Consider the following matrix operations. Choose the correct statement.

    A. Adding a 2×2 matrix W with another 2×2 matrix Z results in a 4×4 matrix $W + Z$.

    B. Multiplying a 4×3 matrix L with a 3×2 matrix M results in a 4×2 matrix LM.

    C. Given two matrices X and Y, the result of XY is always the same as YX.

    D. Adding a 3×4 matrix J with a 4×3 matrix K results in a 3×3 matrix JK.

9. Consider the system of linear equations in three variables x, y, and z:

$$\begin{cases} x + 2y + 3z & = 9 \\ x - y & = 0 \\ 4x + y - 2z & = 1 \end{cases}$$

What is the value of $x \cdot y \cdot z$?

    A. 3

    B. 2

    C. 1

    D. 0

Calculus

10. What is the domain of the function $g(x) = \frac{x^2 - 4}{e^x - 1}$?

    A. $[1, +\infty)$

    B. $\mathbb{R}$

    C. All $\mathbb{R}$ except $x = \{1\}$

    D. All $\mathbb{R}$ except $x = \{0\}$

11. Given $f(x) = \frac{\sin(x-1)}{x^2 - x}$, the vertical asymptotes of f are:

    A. $x = 0$

    B. $x = -1$

    C. $x = 1$ and $x = 0$

    D. None of the other solutions are correct

12. What is the derivative of this function?

$$g(x) = 2\cos^2(3x)$$

    A. $g'(x) = 4\cos(3x)\sin(3x)$

    B. $g'(x) = -4\cos(3x)$

    C. $g'(x) = -12\cos(3x)\sin(3x)$

    D. None of the other solutions are correct

13. What are the points x at which the function f attains a global maximum, where f is the function

$$f(x) = -x^2 e^x?$$

    A. $x = -2$

    B. $x = 2$

    C. $x = 1$

    D. $x = 0$

14. What is the solution of the following definite integral

$$\int_0^\pi x \cos 2x$$

    A. 0

    B. $\pi^2/4$

    C. 1

    D. None of the other solutions are correct

Geometry

15. Let $\vec{a} = (4, 6, 2)$ and let $\vec{b} = (1, 0, -2)$ be two vectors $\in \mathbb{R}^3$. Which of these vectors is perpendicular to both $\vec{a}$ and $\vec{b}$?

    A. $(3, 4, 2)$

    B. $(-12, 10, -3)$

    C. $(-6, 5, -3)$

    D. $(4, 2, 6)$

16. Find the equation of the plane that passes through the point $(1, 1, 1)$ and is perpendicular to the vector $\vec{v} = (1, -1, 3)$.

    A. $x + y + z = 7$

    B. $x - y + 3z = 3$

    C. $x - y + 3z = -1$

    D. $-x + y - 3z = 1$

17. Compute the area of a right triangle which one of its angles is $\alpha = \pi/3$ and the hypotenuse measures 4.

    A. $\sqrt{3}$

    B. $2\sqrt{3}$

    C. 4

    D. 12

Probability and Statistics

18. The mean temperature over 20 days was $15°$ C. The next 10 days the mean temperature was $30°$ C. What is the mean over those 30 days?

    A. $15°$ C

    B. $18°$ C

    C. $20°$ C

    D. $22.5°$ C

19. Consider two events, A and B. Given that $P(A) = 0.4$ and $P(B) = 0.3$, which of the following is impossible?

    A. $P(A \cap B) = P(A) \cdot P(B)$

    B. $P(A \cap B) = 0.3$

    C. $P(A \cup B) = 0.3$

    D. $P(A \cap B) = 0$

20. Which of the following statements is true about a normal distribution $N(0, 1)$?

    A. Its density function is symmetrical with respect to $\mu = 1$

    B. $P(X = 0) = 1$

    C. $P(X < 0) = 0.5$

    D. $P(X < 1) = 0$

Answer Key

Algebra

- Question 1: C. $3 + 29i$

- Question 2: D. $20 \div 5 - 2 \cdot 3 = -2$

- Question 3: B. $9:30$ AM

- Question 4: C. $k = 6$

- Question 5: B. $(3, 6]$

- Question 6: D. 0

Linear Algebra

- Question 7: D. Vector $(5, 2, -6)$ is perpendicular to $\vec{w}$

- Question 8: B. Multiplying a 4×3 matrix L with a 3×2 matrix M results in a 4×2 matrix LM

- Question 9: B. 2

Calculus

- Question 10: D. All $\mathbb{R}$ except $x = \{0\}$

- Question 11: A. $x = 0$

- Question 12: C. $-12 \cos(3x) \sin(3x)$

- Question 13: D. $x = 0$

- Question 14: A. 0

Geometry

- Question 15: C. $(-6, 5, -3)$

- Question 16: B. $x - y + 3z = 3$

- Question 17: B. $2\sqrt{3}$

Probability and Statistics

- Question 18: C. $20°$ C

- Question 19: C. $P(A \cup B) = 0.3$

- Question 20: C. $P(X < 0) = 0.5$